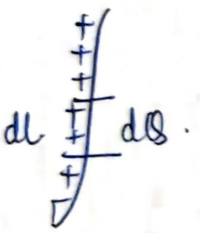
Electrostatics :- It deals with the study of electric fields produced by the charges which are at rest. Here the field produced by static charges is also static.

Types of Charge distribution

1. Point Charge :- If the geometrical dimension of the space over which an electric charge is spread is small compared to the distance at which its effect is evaluated, then it is considered as a point charge. It has only location but not dimension. It can be +ve or -ve.

$\bullet +Q_1$ $\quad$ $\bullet -Q_2$.

2. Line Charge :- If the charges are uniformly distributed on a thin conductor then it is called a line charge.



Consider a thin conductor of length 'L' on which a total charge is Q. Then the ratio $\frac{Q}{L}$ is called line charge density (ρ_L).

Consider an elemental length ΔL on which charge ΔQ, the line charge density is given by

$$\rho_L = \lim_{\Delta L \to 0} \frac{\Delta Q}{\Delta L} \; (C/m). \quad \text{or} \quad \rho_L = \frac{dQ}{dL}.$$

$$\Rightarrow Q = \int \rho_L \cdot dL.$$

3. Surface charge :- Surface charge density denoted by ρ_s is defined as charge per unit surface area measured in C/m^2.

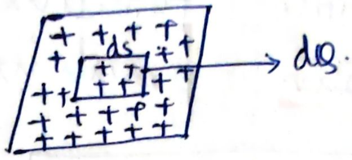
Let dQ be small charge element contained in an elementary surface ds on a surface 'S', then

$$\rho_s = \frac{dQ}{ds} \; (C/m^2).$$

$dQ = \rho_s \, ds$.

or

$$\boxed{Q = \int_s \rho_s \, ds}$$

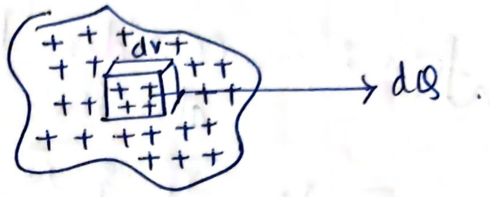


4. Volume Charge :- volume charge density for a volume denoted by ρ_v is defined as charge per unit volume denoted by ρ_v and is measured in C/m^3.

$$\rho_v = \frac{dQ}{dv}.$$

$$dQ = \rho_v \, dv.$$

$$\boxed{Q = \int_v \rho_v \, dv}$$



Module 1.

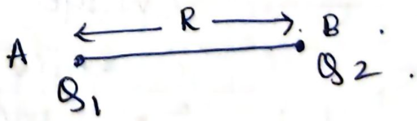
Coulomb's Law :-.

statement :- The electrostatic force F between two point charges Q_1 & Q_2 is

i) directly proportional to the product of the magnitude of the charges.

ii) inversely proportional to the square of the distance between the two charges. &

iii) is directed along the line joining the charge.



ie. $\boxed{F \propto \frac{Q_1 Q_2}{R^2}}$ Newton

where Q_1 & Q_2 are the charges in Coulomb. R is the distance between two charges.

hence, $F = K \frac{Q_1 Q_2}{R^2}$.

K = proportionality constant $= \frac{1}{4\pi \epsilon}$.

& $\epsilon = \epsilon_0 \epsilon_r$.

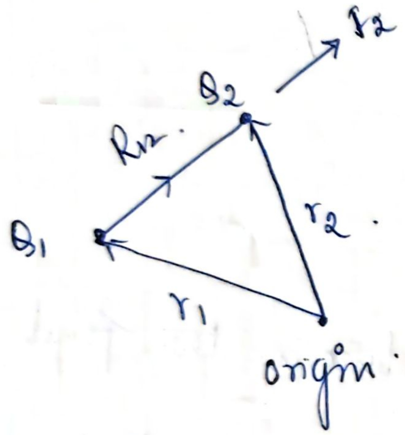
ϵ_0 = Absolute permittivity = 8.854×10^{-12} F/m.
ϵ_r = Relative permittivity = 1 for air or vacuum.

$$\therefore \boxed{F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{R^2}} \text{ Newton}$$

Vector notation

$$\boxed{\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{R^2} \hat{a}_R} \text{ Newton.}$$

Vector form :- Consider two point charges Q_1 & Q_2 located at the points having position vectors $\vec{r_1}$ & $\vec{r_2}$ as in fig.



$R_{12} = r_2 - r_1$; represents the line segment from Q_1 to Q_2 as in fig.

Vector F_2 is the force on Q_2.

$$\vec{F_2} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2} \cdot \hat{a}_{12}$$

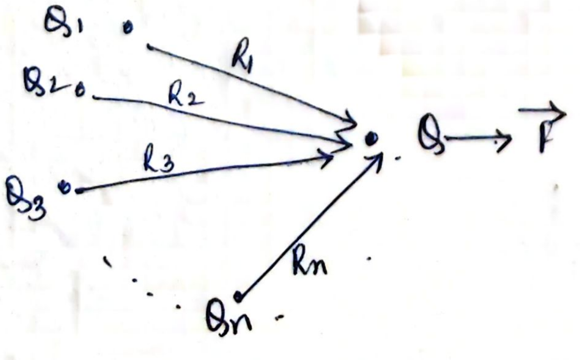
$$\hat{a}_{12} = \frac{\vec{R_{12}}}{|\vec{R_{12}}|} = \frac{\vec{r_2} - \vec{r_1}}{|\vec{r_2} - \vec{r_1}|}.$$

$|\vec{R_{12}}| = R$ = distance between two charges.

According to principle of superposition, the force exerted on a point charge Q by n point charges $Q_1, Q_2, Q_3 \ldots$ is given by the vector sum of the forces exerted by individual charges Q_1 to Q_n on Q ie

$$\vec{F} = \vec{F_1} + \vec{F_2} + \cdots + \vec{F_n}.$$

$$\vec{F_1} = \frac{Q_1 Q}{4\pi\epsilon_0 R_1^2} \hat{a}_{R_1}, \quad \vec{F_2} = \frac{Q_2 Q}{4\pi\epsilon_0 R_2^2} \hat{a}_{R_2}$$



$$\vec{F_3} = \frac{Q_3 Q}{4\pi\epsilon_0 R_3^2} \hat{a}_{R_3} \ \cdots \ \vec{F_n} = \frac{Q_n Q}{4\pi\epsilon_0 R_n^2} \hat{a}_{R_n}$$

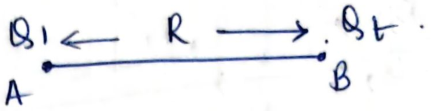
$$\therefore \vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \cdots \vec{F_n}$$

$$\vec{F} = \frac{Q}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{Q_i}{R_i^2} \hat{a}_{R_i}$$

Electric field Intensity :- E.F.I at any point in an electric field is the force experienced by a unit positive charge placed at that point.

Consider a charge Q_1 in fixed position, second charge a test charge Q_t.
The force on it is given by coulomb's law.

$$\vec{F}_t = \frac{Q_1 Q_t}{4\pi\epsilon_0 R_{1t}^2}\,\hat{a}_{1t}$$



Force per unit charge is given by.

$$\frac{F_t}{Q_t} = \frac{Q_1}{4\pi\epsilon_0 R_{1t}^2}\,\hat{a}_{1t}$$ which defines the electric field intensity

$$E = \frac{F_t}{Q_t}$$

$$\boxed{\vec{E} = \frac{Q}{4\pi\epsilon_0 R_{1t}^2}\,\hat{a}_{1t}}$$

In general, $\boxed{\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2}\,\hat{a}_R}$ volt/m or N/c.

Electric field due to array of point charges:

This can be found by employing the principle of superposition.
It states that,
" The force between any two charges is independent of the presence of other charges due to which the effect of individual charges could be superposed."

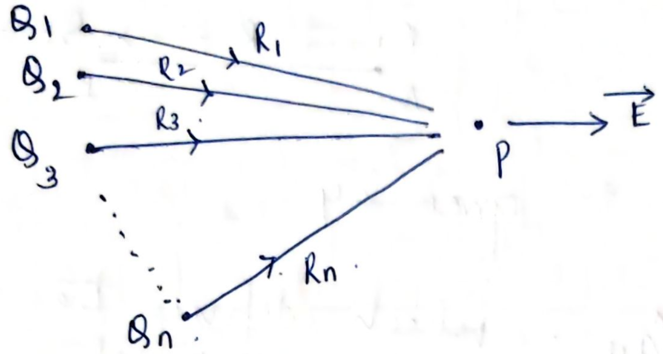
Consider an array of point charges $Q_1, Q_2, \dots Q_n$ at distances $R_1, R_2 \dots R_n$ from a point P.
The total field $\vec{E}$ at P due to all the charges can be written by using the principle of superposition as,

$$\vec{E} = \vec{E_1} + \vec{E_2} + \cdots + \vec{E_n}$$

$$= \frac{Q_1}{4\pi\epsilon_0 R_1^2}\hat{a}_{R1} + \frac{Q_2}{4\pi\epsilon_0 R_2^2}\hat{a}_{R2} + \cdots + \frac{Q_n}{4\pi\epsilon_0 R_n^2}\hat{a}_{Rn}.$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{Q_i}{R_i}\hat{a}_{Ri}$$

$$\hat{a}_{Ri} = \frac{\vec{r_p} - \vec{r_i}}{|\vec{r_p} - \vec{r_i}|}.$$



where, $\vec{r_p}$ = position vector of point P.

$\vec{r_i}$ = position vector of point where charge Q_i is placed

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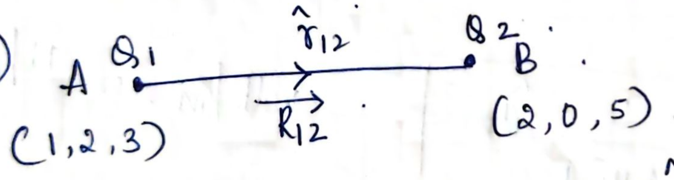
1. A charge $Q_1 = 3\times10^{-4}$ C is located at M(1,2,3) and a charge $Q_2 = -10^{-4}$ C is located at (2,0,5).
   i) find the force exerted on Q_2 by Q_1.
   ii) find the force exerted on Q_1 by Q_2.

:- i)



$$\hat{r}_{12} = \frac{\vec{R_{12}}}{|\vec{R_{12}}|} = \frac{(2-1)\hat{a}_x + (0-2)\hat{a}_y + (5-3)\hat{a}_z}{\sqrt{(2-1)^2 + (0-2)^2 + (5-3)^2}}.$$

$$= \frac{\hat{a}_x - 2\hat{a}_y + 2\hat{a}_z}{\sqrt{1^2+4+4}} = \frac{\hat{a}_x - 2\hat{a}_y + 2\hat{a}_z}{3}.$$

$$\vec{F_2} = \frac{Q_1 Q_2}{4\pi\epsilon_0 \cdot R_{12}^2}\hat{r}_{12} = \frac{(3\times10^{-4})(-10^{-4})}{4\pi\times8.854\times10^{-12}\times3^2}\times\left[\frac{\hat{a}_x - 2\hat{a}_y + 2\hat{a}_z}{3}\right]$$

$$\vec{F_2} = -9.99\hat{a}_x + 19.98\hat{a}_y - 19.98\hat{a}_z$$

$$|\vec{F_2}| = \sqrt{(-9.99)^2 + (19.98)^2 + (-19.98)^2} = 29.97\,\text{N}$$

$$\simeq \underline{30\,\text{N}}.$$

b) $\hat{r}_{21} = \dfrac{\vec{R}_{21}}{|\vec{R}_{21}|} = \dfrac{(1-2)\hat{a}_x + (2-0)\hat{a}_y + (3-5)\hat{a}_z}{\sqrt{(1-2)^2 + (2-0)^2 + (3-5)^2}}$ (10)

$$\hat{r}_{21} = \frac{-\hat{a}_x + 2\hat{a}_y - 2\hat{a}_z}{\sqrt{1+4+4}} = \frac{-\hat{a}_x + 2\hat{a}_y - 2\hat{a}_z}{3}$$

$$\vec{F}_1 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{21}^2}\hat{r}_{21} = \frac{(3\times10^{-4})(-10^{-4})}{4\pi\times8.854\times10^{-12}\times3^2}\left[\frac{-\hat{a}_x + 2\hat{a}_y - 2\hat{a}_z}{3}\right]$$

$$\vec{F}_1 = 9.99\hat{a}_x - 19.98\hat{a}_y + 19.98\hat{a}_z$$

$$|\vec{F}_1| = \sqrt{(9.99)^2 + (-19.98)^2 + (19.98)^2} = 29.97N \simeq 30N$$

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2. A charge $Q_1 = -20\mu C$ located at $P(-6,4,6)$ & a charge $Q_2 = 50\mu C$ at $(5,8,-2)$. Find the force exerted on Q_2 by Q_1.

:-

$$\underset{(-6,4,6)}{Q_1} \xrightarrow[\hat{r}_{12}]{R_{12}} \underset{(5,8,-2)}{Q_2}$$

$$\hat{r}_{12} = \frac{\vec{R}_{12}}{|\vec{R}_{12}|} = \frac{(5+6)\hat{a}_x + (8-4)\hat{a}_y + (-2-6)\hat{a}_z}{\sqrt{11^2+4^2+8^2}}$$

$$\hat{r}_{12} = \frac{11\hat{a}_x + 4\hat{a}_y - 8\hat{a}_z}{14.1774}$$

$$\vec{F}_2 = \frac{(-20\times10^{-6})(50\times10^{-6})}{4\pi\times8.854\times10^{-12}\times(14.1774)^2}\times\frac{11\hat{a}_x + 4\hat{a}_y - 8\hat{a}_z}{14.1774}$$

$$\vec{F}_2 = -0.0346\hat{a}_x - 0.0126\hat{a}_y + 0.0252\hat{a}_z \ N$$

$$|\vec{F}_2| = 44.634\ mN$$

3. A point charge $Q_1 = 250\mu C$ is located at $(2,-1,-3)$ experiences a force $\vec{F} = 4\hat{a}_x - 4\hat{a}_y + 8\hat{a}_z$ due to a point charge Q_2 at $(3,-2,-1)$. Determine Q_2.

:- $$\vec{r}_{21} = \frac{\vec{R}_{21}}{|\vec{R}_{21}|} = \frac{(2-3)\hat{a}_x + (-1+2)\hat{a}_y + (-3+1)\hat{a}_z}{\sqrt{(-1)^2+(1)^2+(-2)^2}}$$

$$\vec{r}_{21} = \frac{-\hat{a}_x + \hat{a}_y - 2\hat{a}_z}{\sqrt{6}}$$

$$\vec{F}_1 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{21}^2} \times \vec{r}_{21}$$

$$(4\hat{a}_x - 4\hat{a}_y + 8\hat{a}_z) = \frac{250\times10^{-6}\, Q_2}{4\pi\times8.85\times10^{-12}\times6}\left\{\frac{-\hat{a}_x+\hat{a}_y-2\hat{a}_z}{\sqrt{6}}\right\}$$

$$4\left(\hat{a}_x - \cancel{\hat{a}_y} + 2\hat{a}_z\right) = \frac{(250\times10^{-6})(Q_2)}{4\pi\times8.854\times10^{-6}\times6}\left(\frac{-1}{\sqrt{6}}\right)\left(\hat{a}_x - \cancel{\hat{a}_y} + 2\hat{a}_z\right)$$

$$\boxed{Q_2 = -2.616\times10^{-5}\ C}$$

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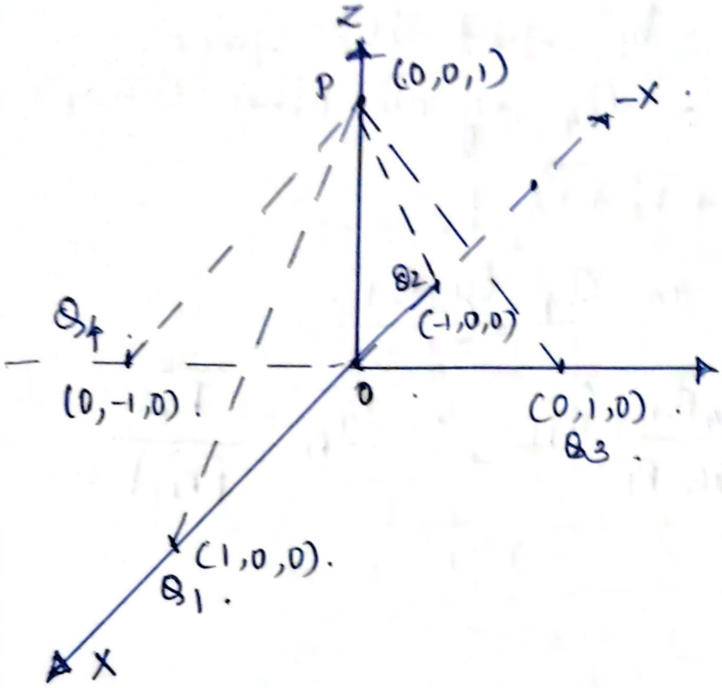
4. Four point charges each of $10\mu C$ are placed in free space at points $(1,0,0)$, $(-1,0,0)$ $(0,1,0)$ & $(0,-1,0)$ m respectively. Determine the force on a point charge of $30\mu C$ located at a point $(0,0,1)$ m.

:- $$\vec{F} = \vec{F}_{1P} + \vec{F}_{2P} + \vec{F}_{3P} + \vec{F}_{4P}$$



$$\vec{F_1} = \frac{(10\times10^{-6})(30\times10^{-6})}{4\pi\times8.854\times10^{-12}} \cdot \frac{\hat{r}_{1P}}{R_{1P}^2}$$

$$\hat{r}_{1P} = \frac{\vec{R_{1P}}}{|\vec{R_{1P}}|}$$

$$= \frac{(0-1)\hat{a}_x + (0-0)\hat{a}_y + (1-0)\hat{a}_z}{\sqrt{2}}$$

$$\hat{r}_{1P} = \frac{-\hat{a}_x + \hat{a}_z}{\sqrt{2}}$$

$$\vec{F_1} = \frac{2.6975}{2} \times \frac{(-\hat{a}_x + \hat{a}_z)}{\sqrt{2}}$$

$$\vec{F_1} = 0.953(-\hat{a}_x + \hat{a}_z)$$

$$\vec{F_2} = \frac{2.6975}{2} \times \frac{(\hat{a}_x + \hat{a}_z)}{\sqrt{2}} = 0.953(\hat{a}_x + \hat{a}_z)$$

$$\vec{F_3} = \frac{2.6975}{2} \times \frac{(-\hat{a}_y + \hat{a}_z)}{\sqrt{2}} = 0.953(-\hat{a}_y + \hat{a}_z)$$

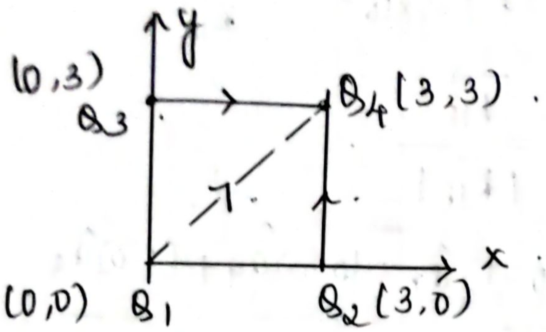
$$\vec{F_4} = \frac{2.6975}{2} \times \frac{(\hat{a}_y + \hat{a}_z)}{\sqrt{2}} = 0.953(\hat{a}_y + \hat{a}_z)$$

$$\vec{F} = 0.953\{-\hat{a}_x + \hat{a}_z + \hat{a}_x + \hat{a}_z - \hat{a}_y + \hat{a}_z + \hat{a}_y + \hat{a}_z\}$$

$$\boxed{\vec{F} = 3.813\hat{a}_z \text{ N}}$$

5. Four identical charges 8μC are at the 4 corners of the square 3m at in length. Find the force acting on the charge Q_4. If the 4 charges are respectively Q_1, Q_2, Q_3, Q_4.

: Soln :



we need to find the force exerted on Q_4 by all other charges.

$$\vec{F_4} = \vec{F_1} + \vec{F_2} + \vec{F_3}$$

i) force on Q_4 by Q_1

$$\vec{F_1} = \frac{Q_4 Q_1}{4\pi\epsilon_0 R_1^2}\hat{a}_{R_1} \quad ; \quad \hat{a}_{R_1} = \frac{\vec{R_1}}{|\vec{R_1}|}$$

$$\vec{R_1} = 3\hat{a}_x + 3\hat{a}_y$$

$$|\vec{R_1}| = \sqrt{3^2+3^2} = 3\sqrt{2}$$

$$\therefore \vec{F_1} = \frac{Q_4 Q_1}{4\pi\epsilon_0} \times \frac{(3\hat{a}_x + 3\hat{a}_y)}{18 \times 3\sqrt{2}}$$

ii) force on Q_4 by Q_2

$$\vec{F_2} = \frac{Q_4 Q_2}{4\pi\epsilon_0 R_2^2}\hat{a}_{R_2} \quad ; \quad \hat{a}_{R_2} = \frac{\vec{R_2}}{|\vec{R_2}|}$$

$$\vec{R_2} = (3-3)\hat{a}_x + (3-0)\hat{a}_y = 3\hat{a}_y$$

$$|\vec{R_2}| = 3$$

$$\therefore \vec{F_2} = \frac{Q_4 Q_2}{4\pi\epsilon_0} \times \left\{\frac{3\hat{a}_y}{9 \times 3}\right\}$$

iii) force on Q_4 by Q_3

$$\vec{F_3} = \frac{Q_4 Q_3}{4\pi\epsilon_0 R_3^2}\hat{a}_{R_3} \quad ; \quad \hat{a}_{R_3} = \frac{\vec{R_3}}{|\vec{R_3}|}$$

$$\vec{R_3} = (3-0)\hat{a}_x + (3-3)\hat{a}_y = 3\hat{a}_x$$

$$|\vec{R_3}| = 3$$

$$\therefore \vec{F_3} = \frac{Q_4 Q_3}{4\pi\epsilon_0} \times \left\{ \frac{3\hat{a}_x}{3^2 \times 3} \right\}.$$

$$\therefore \vec{F_4} = \vec{F_1} + \vec{F_2} + \vec{F_3} \quad ; \quad \text{given } Q_1 = Q_2 = Q_3 = Q_4 = 8\mu C$$

$$= \frac{Q_4 Q}{4\pi\epsilon_0} \left\{ \frac{3\hat{a}_x + 3\hat{a}_y}{76.36} + \frac{3\hat{a}_y}{27} + \frac{3\hat{a}_x}{27} \right\}.$$

$$= \frac{(8\times10^{-6})(8\times10^{-6})}{4\pi\times8.854\times10^{-12}} \left\{ 0.039\hat{a}_x + 0.039\hat{a}_y + 0.111\hat{a}_y + 0.111\hat{a}_x \right\}.$$

$$= 0.5755 \left\{ 0.15\hat{a}_x + 0.15\hat{a}_y \right\}.$$

$$\boxed{\vec{F_4} = 0.086\,\hat{a}_x + 0.086\,\hat{a}_y}$$

6) Three point charges each of 'q' C are located at the corners of an equidateral triangle of side 'd' meters. Determine the magnitude & direction of force on each of these charges.

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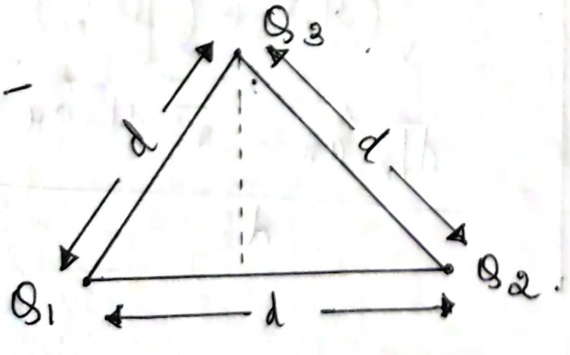

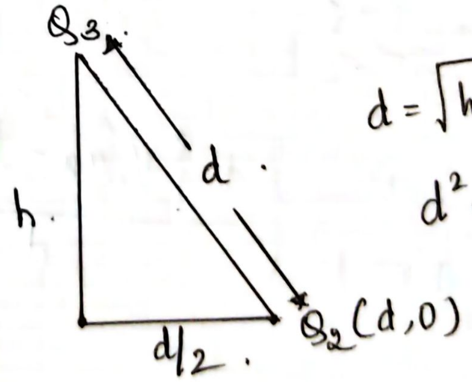

$$d = \sqrt{h^2 + (d/2)^2}$$

$$d^2 = h^2 + \frac{d^2}{4}.$$

$$\boxed{h = \sqrt{3}\,\frac{d}{2}}$$

$\therefore$ co-ordinates of all three points are.

$Q_1\ (0,0)$

$Q_2\ (d,0)$

$Q_3\ (d/2, \sqrt{3}\, d/2)$.

Force on Q_2 is

$$\vec{F_2} = \vec{F_{12}} + \vec{F_{32}}$$

$$\vec{F_{12}} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\hat{a}_{R12} \quad ; \quad \hat{a}_{R12} = \frac{\vec{R_{12}}}{|\vec{R_{12}}|} = \frac{\vec{R_2} - \vec{R_1}}{|\vec{R_{12}}|}$$

$$= \frac{(d-0)\hat{a}_x + (0-0)\hat{a}_y}{d}$$

$$= \frac{Q_1 Q_2}{4\pi\epsilon_0 \times d^2}\left\{\frac{d\hat{a}_x}{d}\right\}$$

$$\boxed{\vec{F_{12}} = \frac{Q_1 Q_2}{4\pi\epsilon_0 d^2}\hat{a}_x}$$

$$\vec{F_{32}} = \frac{Q_3 Q_2}{4\pi\epsilon_0 R_{32}^2}\hat{a}_{R32} \quad ; \quad \hat{a}_{R32} = \frac{\vec{R_{32}}}{|\vec{R_{32}}|} = \frac{\vec{R_2} - \vec{R_3}}{|\vec{R_{32}}|}$$

$$= \cancel{\frac{(d - d/2)^2\hat{a}_x + (\frac{\sqrt{3}d}{2} - 0)\hat{a}_y}{\sqrt{(d/2)^2 + 3(d/2)^2}}}$$

$$= \cancel{\frac{d^2/4\,\hat{a}_x + 3d^2/4\,\hat{a}_y}{}}$$

$$= \frac{(d - d/2)\hat{a}_x + (0 - \sqrt{3}\frac{d}{2})\hat{a}_y}{\sqrt{(d/2)^2 + (-\sqrt{3}d/2)^2}}$$

$$= \frac{d/2\,\hat{a}_x - \sqrt{3}\,d/2\,\hat{a}_y}{d}$$

$$\vec{F_{32}} = \frac{Q_3 Q_2}{4\pi\epsilon_0 d^2}\left\{\frac{d/2\,\hat{a}_x - \sqrt{3}\frac{d}{2}\hat{a}_y}{d}\right\}$$

$$\cancel{|\vec{E_{32}}| = \frac{Q^2}{4\pi\epsilon_0 d^2}\{}$$

$$\vec{F_{32}} = \frac{Q_3 Q_2}{4\pi\epsilon_0 d^2}\left\{\frac{d\{1/2\,\hat{a}_x - \frac{\sqrt{3}}{2}\hat{a}_y\}}{d}\right\}$$

$$\vec{F_2} = \frac{Q_1 Q_2}{4\pi\epsilon_0 d^2}\hat{a}_x + \frac{Q_3 Q_2}{4\pi\epsilon_0 d^2}\left\{1/2\,\hat{a}_x - \frac{\sqrt{3}}{2}\hat{a}_y\right\}.$$

Since $Q_1 = Q_2 = Q_3$.

$$= \frac{Q_1^2}{4\pi\epsilon_0 d^2}\left\{\hat{a}_x + 1/2\,\hat{a}_x - \frac{\sqrt{3}}{2}\hat{a}_y\right\} = \frac{Q^2}{4\pi\epsilon_0 d^2}\left\{3/2\,\hat{a}_x - \frac{\sqrt{3}}{2}\hat{a}_y\right\}$$

$$|\vec{F_2}| = \frac{Q^2}{4\pi\epsilon_0 d^2}\sqrt{\frac{9}{4}+\frac{3}{4}}$$

$$\boxed{|\vec{F_2}| = \frac{Q^2}{4\pi\epsilon_0 d^2}\sqrt{3}}$$

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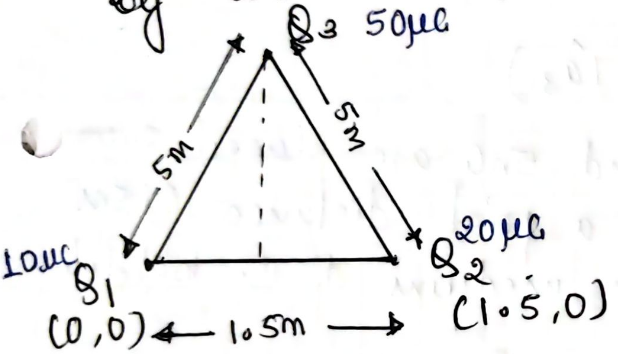
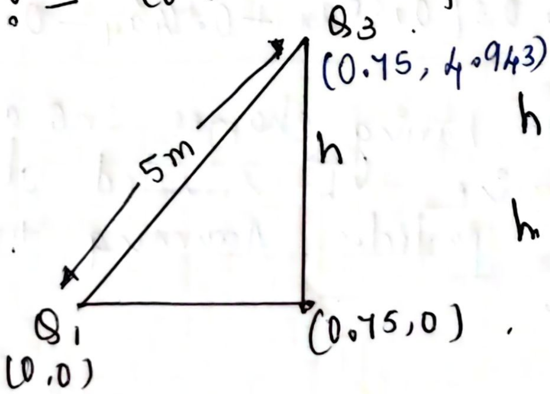
(7) Three point charges are located at corners of an isoceles Δ^{le} as shown. find the force experienced by the charge Q_2.



:- Co-ordinates of Q_3.



$$h = \sqrt{5^2 - 0.75^2}$$

$$h = 4.943$$

$$\vec{F_2} = \vec{F_{12}} + \vec{F_{32}}$$

$$\vec{F_{12}} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\hat{a}_{R_{12}}$$

$$\vec{F_{12}} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\frac{\vec{R_{12}}}{|\vec{R_{12}}|}$$

$$= \frac{(10\times10^{-6})(20\times10^{-6})}{4\pi\times8.854\times10^{-12}\times1.5^2}\frac{(1.5\hat{a}_x)}{1.5}$$

$$\vec{F_{12}} = 0.799\hat{a}_x$$

$$\vec{F_{32}} = \frac{Q_3 Q_2}{4\pi\epsilon_0 R_{32}^2}\hat{a}_{R_{32}}$$

$$= \frac{Q_3 Q_2}{4\pi\epsilon_0 R_{32}^2}\frac{\vec{R_{32}}}{|\vec{R_{32}}|}$$

$$= \frac{50\times10^{-6}\times20\times10^{-6}}{4\pi\times8.854\times10^{-12}\times5^2}\frac{(0.75\hat{a}_x + 4.9\hat{a}_y)}{\sqrt{0.75^2+4.9^2}}$$

$$\vec{F_{32}} = \cancel{0.849\hat{a}_x} - 0.355\hat{a}_y$$

$$\vec{F_{32}} = 0.053\hat{a}_x - 0.355\hat{a}_y$$

$$\vec{F_2} = \vec{F_{12}} + \vec{F_{32}}$$

$$= 0.799\hat{a}_x + 0.053\hat{a}_x - 0.355\hat{a}_y$$

$$\vec{F_2} = 0.849\hat{a}_x - 0.355\hat{a}_y$$

(8) Determine the electric field intensity at (3, 4, −2) due to a 3μC charge placed at the origin in free space.

:−

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2}\hat{a}_R$$

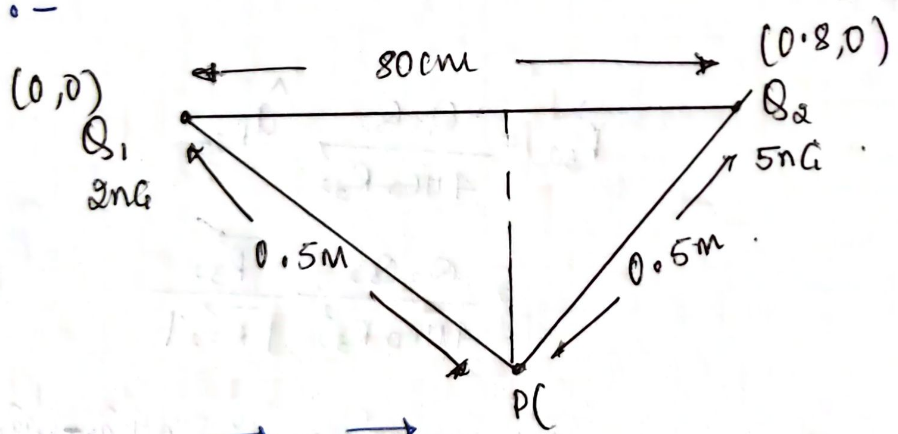
$$= \frac{3\times10^{-6}}{4\pi\times8.854\times10^{-12}\times29}\;\frac{3\hat{a}_x + 4\hat{a}_y - 2\hat{a}_z}{\sqrt{29}}$$

$$\vec{E} = 931.03(0.55\hat{a}_x + 0.74\hat{a}_y - 0.37\hat{a}_z)$$

---

(9) Two particles having charges 2nC and 5nC are placed 80cm apart. Determine $\vec{E}$ situated at a point distance 0.5m from each particles. Assuming the medium to be bakelite with $\epsilon_r = 5$.

:−



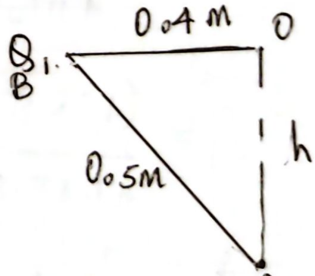
From right angle Triangle.

$$BC^2 = OB^2 + OC^2$$

$$0.5^2 = 0.4^2 + OC^2$$

$$OC^2 = 0.5^2 - 0.4^2$$

$$\boxed{OC = 0.3}$$



$$\vec{E} = \vec{E_1} + \vec{E_2}$$

$$\vec{E_1} = \frac{Q_1}{4\pi\epsilon R_1^2}\hat{a}_{R_1}, \quad \vec{E_2} = \frac{Q_2}{4\pi\epsilon R_2^2}\hat{a}_{R_2}$$

$\epsilon = \epsilon_0\epsilon_r$ [$\epsilon_r = 5$ given].

$$\hat{a}_{R_1} = \frac{\vec{R_1}}{|\vec{R_1}|}, \quad Q_3(0.4, 0.3),\ Q_1(0,0).$$

$$\vec{R_1} = (0.4)\hat{a}_x + 0.3\hat{a}_y$$

$$|\vec{R_1}| = \sqrt{0.4^2 + 0.3^2} = 0.5$$

$\vec{R_2}$; $Q_3(0.4, 0.3)$ $Q_2(0.8, 0)$

$$\vec{R_2} = (0.4 - 0.8)\hat{a}_x + 0.3\hat{a}_y$$

$$\vec{R_2} = -0.4\hat{a}_x + 0.3\hat{a}_y$$

$$|\vec{R_2}| = 0.5$$

$$\vec{E} = \vec{E_1} + \vec{E_2}$$

$$\vec{E} = \frac{Q_1}{4\pi\epsilon R_1^2}\hat{a}_{R_1} + \frac{Q_2}{4\pi\epsilon R_2^2}\hat{a}_{R_2}$$

$$= \left\{\frac{2\times10^{-9}}{4\pi\times8.854\times10^{-12}\times5\times(0.5)^2}\times\left[\frac{0.4\hat{a}_x+0.3\hat{a}_y}{0.5}\right]\right\} +$$

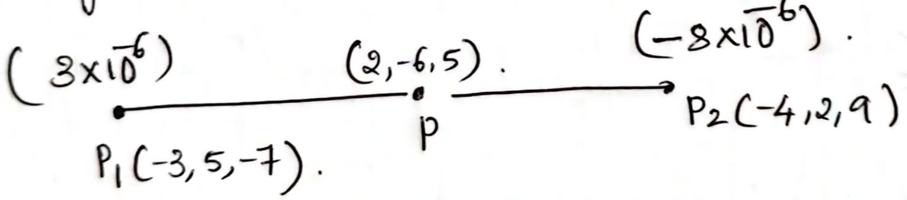
$$\left\{\frac{5\times10^{-9}}{4\pi\times8.854\times10^{-12}\times5\times0.5^2}\times\left[\frac{-0.4\hat{a}_x+0.3\hat{a}_y}{0.5}\right]\right\}$$

$$\vec{E} = -17232\hat{a}_x + 30.15\hat{a}_y \ (\text{V/m})$$

---

(10) Two point charges of magnitudes $3\mu C$ and $-8\mu C$ are located at places $P_1(-3,5,-7)$ and $P_2(-4,2,9)$ respectively in free space. Evaluate the electric field and also its magnitude at the point $P(2,-6,5)$.

:—



The field E_1 at P due to the charge at P_1 is

$$\vec{E_1} = \frac{3\times10^{-6}}{4\pi\epsilon_0 R_1^2}\hat{a}_{R_1}$$

$$\hat{a}_{R_1} = \frac{\vec{R_1}}{|\vec{R_1}|} = \frac{[2-(-3)]\hat{a}_x + (-6-5]\hat{a}_y + [5-(-7)]\hat{a}_3}{\sqrt{5^2+(-11)^2+(12)^2}}$$

$$\hat{a}_{R_1} = 0.29\hat{a}_x - 0.65\hat{a}_y + 0.7\hat{a}_3$$

$$|\vec{R_1}| = \sqrt{5^2+(-11)^2+(12)^2} = 17.029$$

$$\vec{E_1} = \frac{3\times10^{-6}}{4\pi\times8.854\times10^{-12}\times17.029^2}\times\left[0.29\hat{a}_x - 0.65\hat{a}_y + 0.7\hat{a}_3\right]$$

$$\vec{E_1} = 26.98\hat{a}_y - 60.045\hat{a}_y + 65.1\hat{a}_3$$

The field at P due to a charge at P_2 is

$$\vec{E_2} = \frac{-8\times10^{-6}}{4\pi\epsilon_0 R_2^2}\hat{a}_{R_2}$$

$$\hat{a}_{R_2} = \frac{\vec{R_2}}{|\vec{R_2}|} = \frac{[2-(-4)]\hat{a}_x + [-6-2]\hat{a}_y + (5-9)\hat{a}_3}{\sqrt{6^2+(-8)^2+(-4)^2}}$$

$$\hat{a}_{R_2} = \frac{6\hat{a}_x - 8\hat{a}_y - 4\hat{a}_3}{10.77}$$

$$\vec{E_2} = \frac{-8\times10^{-6}}{4\pi\times8.854\times10^{-12}\times\cancel{10.77^2}\,116}\times\left(\frac{6\hat{a}_x - 8\hat{a}_y - 4\hat{a}_3}{10.77}\right)$$

$$\vec{E_2} = -345\hat{a}_x - 460\hat{a}_y - 229\hat{a}_3$$

$$\vec{E} = \vec{E_1} + \vec{E_2}$$

$$\vec{E} =$$

$$|\vec{E}| =$$

$$\hat{a}_E = \frac{\vec{E}}{|\vec{E}|} =$$

(0,0)

(11). A charge of 1C is at (2,0,0). What charge must be placed at (-2,0,0) which will make 'y' component of total $\vec{E}$ zero at the point (1,2,2). (15)

:- $\vec{E} = \vec{E_A} + \vec{E_B}$

$$= \frac{1}{4\pi\epsilon_0}\left\{\frac{Q_A}{R_A^2}\hat{a}_{RA} + \frac{Q_B}{R_B^2}\hat{a}_{RB}\right\}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\left[\left\{\frac{1\times[(1-2)\hat{a}_x + (2-0)\hat{a}_y + (2-0)\hat{a}_z]}{3^2 \times \sqrt{(-1)^2+2^2+2^2}}\right\} + \left\{\frac{Q_2 \times [(1+2)\hat{a}_x + (2-0)\hat{a}_y + (2-0)\hat{a}_z]}{17 \times \sqrt{3^2+2^2+2^2}}\right\}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left\{\frac{-\hat{a}_x + 2\hat{a}_y + 2\hat{a}_z}{27} + \frac{Q_2(3\hat{a}_x + 2\hat{a}_y + 2\hat{a}_z)}{70.0927}\right\}$$

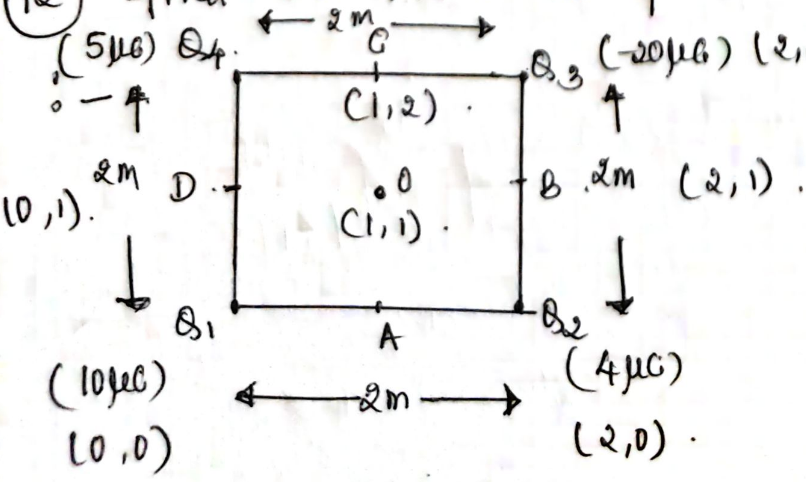
the 'y' component of $\vec{E}$ must be zero.

$$\frac{2}{27} + \frac{2Q_2}{70.0927} = 0.$$

$$\boxed{Q_2 = -2.596\ C}$$

---

(12). Find the E.F.I at point A, B, C, D & O. :-



$$\vec{E_A} = \vec{E_{1A}} + \vec{E_{2A}} + \vec{E_{3A}} + \vec{E_{4A}}$$

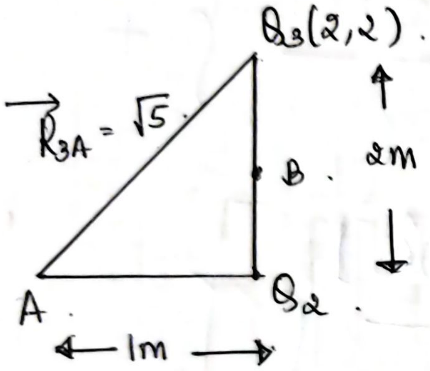
$$\vec{E_{1A}} = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{Q_1}{R_{1A}^2}\hat{a}_{1A}$$

$$\hat{a}_{1A} = \frac{\vec{R_{1A}}}{|\vec{R_{1A}}|}$$

$$\vec{E}_{1A} = \frac{(9\times10^{9})(10\times10^{-6})\,\hat{a}_x}{1} = 90\times10^{3}\,\hat{a}_x$$

$$\vec{E}_{2A} = \frac{(9\times10^{9})(4\times10^{-6})\cdot(-\hat{a}_x)}{1} = -36\times10^{3}\,\hat{a}_x$$

$$\vec{E}_{3A} = \frac{(9\times10^{9})(-20\times10^{-6})\cdot(-\hat{a}_x - 2\hat{a}_y)}{5\times\sqrt{5}} = \frac{36\times10^{3}}{\sqrt{5}}(\hat{a}_x + 2\hat{a}_y)$$



$$\vec{E}_{4A} = \frac{(9\times10^{9})(5\times10^{-6})}{5}\,\frac{(\hat{a}_x - 2\hat{a}_y)}{\sqrt{5}}$$

$$\vec{E}_{4A} = \frac{9\times10^{3}}{\sqrt{5}}(\hat{a}_x - 2\hat{a}_y)$$

$$\therefore \vec{E}_{A\,total} = \vec{E}_{1A} + \vec{E}_{2A} + \vec{E}_{3A} + \vec{E}_{4A}$$

$$= (90\times10^{3})\hat{a}_x - (36\times10^{3})\hat{a}_x + \left(\frac{36\times10^{3}}{\sqrt{5}}\right)\hat{a}_x + \left(\frac{72\times10^{3}}{\sqrt{5}}\right)\hat{a}_y + \left(\frac{9\times10^{3}}{\sqrt{5}}\right)\hat{a}_x - \left(\frac{18\times10^{3}}{\sqrt{5}}\right)\hat{a}_y$$

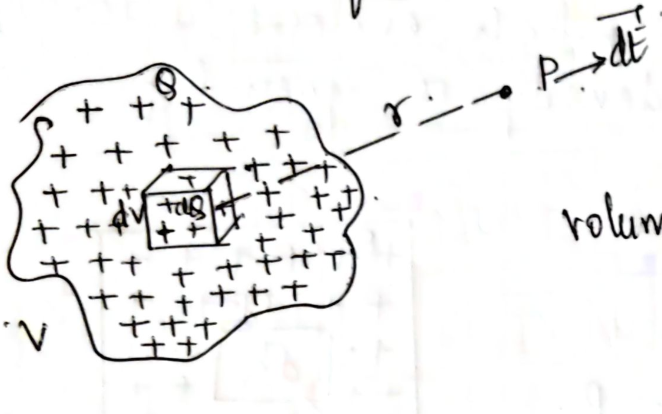
$=$

$|\vec{E}| =$

Field due to Continuous Volume Charge Distribution

(16)

" If the charge distribution is such that the charges are distributed continuously in a volume then it is referred to as volume charge distribution"



If 'Δq' is the charge in a small volume Δv then volume charge density

$$\rho_v = \left[\frac{\Delta q}{\Delta v}\right]_{\Delta v \to 0}$$

or

$$\rho_v = \frac{dq}{dv} \quad (C/m^3)$$

The electric field at point 'p' due to the point charge 'dq' is

$$\vec{dE} = \frac{dq}{4\pi\epsilon_0 r^2}\,\hat{a}_r$$

Since charge distribution is continuous, the field at 'p' due to the entire volume is.

$$\vec{E} = \int \vec{dE} = \int_v \frac{dq}{4\pi\epsilon_0 r^2}\hat{a}_r = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^2}\hat{a}_r$$

$$dq = \rho_v\, dv$$

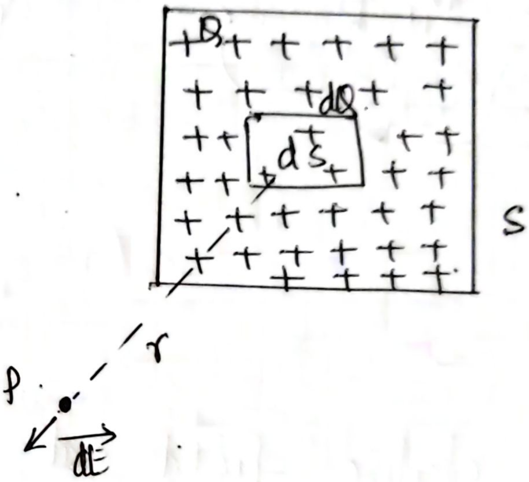
$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\int_v \frac{\rho_v\, dv}{r^2}\hat{a}_r} \quad V/m$$

Electric Field due to Surface Charge Distribution.

Charge distribution is such that the charges are continuously distributed on a two dimensional surface, then it is referred to as surface charge distribution or a sheet of charge.

Consider a small surface ds which encloses a charge dQ. the surface charge density is given by

$$\rho_s = \frac{dQ}{ds} \quad (C/m^2).$$



The electric field at point P. due to charge dQ is

$$\vec{dE} = \frac{dQ}{4\pi\epsilon_0 r^2} \hat{a}_r$$

Since charge distribution is continuous, the field at P due to the entire surface is

$$\vec{E} = \int \vec{dE} = \int \frac{dQ}{4\pi\epsilon_0 r^2} \hat{a}_r$$

but $dQ = \rho_s ds$.

$$\therefore \vec{E} = \int \frac{\rho_s ds}{4\pi\epsilon r^2} \hat{a}_r$$

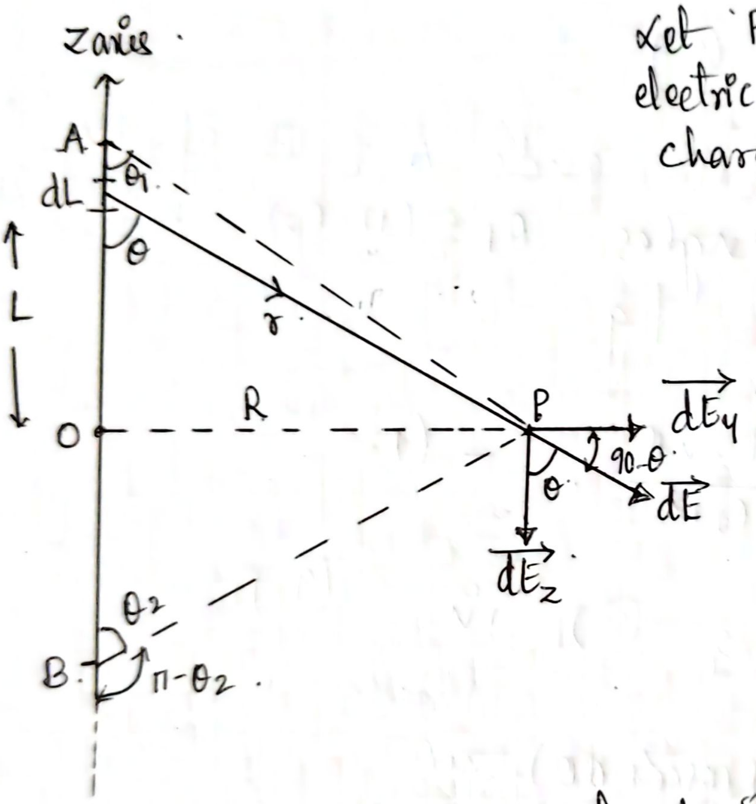
$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon} \int_S \frac{\rho_s ds}{r^2} \hat{a}_r} \quad V/m.$$

Electric field due to a finite line charge

Let AB be a straight line charge along the z-axis. Let its linear charge density be ρ_L. If 'dq' is the charge over a length dL, then

$$\rho_L = \frac{dq}{dL}$$

$$\Rightarrow \boxed{dq = \rho_L\, dL} \quad \text{—(1)}$$



Let 'P' be the point at which the electric field due to the line charge is required.

Let PO = R

The electric field $\vec{dE}$ at P due to incremental length is given by

$$\boxed{\vec{dE} = \frac{dq}{4\pi\epsilon_0 r^2}\hat{a}_r} \quad \text{—(2)}$$

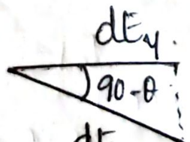
$\vec{dE}$ can be resolved into two rectangular components $\vec{dE_y}$ and $\vec{dE_z}$.

$$\therefore \vec{dE} = \vec{dE_y} + \vec{dE_z} \quad \text{—(3)}$$

$$\vec{dE_y} = dE_y\,\hat{a}_y \quad \text{—(4)}$$

$$\vec{dE_z} = dE_z(-\hat{a}_z) \quad \text{—(5)}$$

From triangle,



$$\cos(90-\theta) = \frac{dE_y}{dE}$$

$$dE_y = dE\sin\theta$$

$$dE_y = \frac{dQ}{4\pi\epsilon_0 r^2}\sin\theta$$

from eq (1), $dq = \rho_L \, dl$.

$$\therefore dE_y = \frac{\rho_L \, dl}{4\pi\epsilon_0 r^2} \sin\theta \quad \text{--- (6)}$$

from fig,

$$\cot\theta = \frac{L}{R} \Rightarrow L = R\cot\theta$$

$$\boxed{dl = -R\,\mathrm{cosec}^2\theta \, d\theta} \quad \text{--- (7)}$$

& $$\boxed{r = R\,\mathrm{cosec}\,\theta} \quad \text{--- (8)}$$

Let the line joining the ends A & B of the line charge to P makes angles θ_1 & $(\pi - \theta_2)$ with the line charge as in fig.

$$\therefore E_y = \int_{\pi-\theta_2}^{\theta_1} \frac{\rho_L \, dl \sin\theta}{4\pi\epsilon_0 r^2} \quad \text{--- (9)}$$

Substitute (7) & (8) in (9)

$$E_y = \int_{\pi-\theta_2}^{\theta_1} \frac{\rho_L(-R\,\mathrm{cosec}^2\theta \, d\theta)\sin\theta}{4\pi\epsilon_0 (R\,\mathrm{cosec}\,\theta)^2}$$

$$E_y = \int_{\pi-\theta_2}^{\theta_1} -\frac{\rho_L}{4\pi\epsilon_0 R} \sin\theta \, d\theta$$

$$= -\frac{\rho_L}{4\pi\epsilon_0 R} \int_{\pi-\theta_2}^{\theta_1} \sin\theta \, d\theta$$

$$= -\frac{\rho_L}{4\pi\epsilon_0 R} (-\cos\theta)_{\pi-\theta_2}^{\theta_1} = \frac{\rho_L}{4\pi\epsilon_0 R} (\cos\theta)_{\pi-\theta_2}^{\theta_1}$$

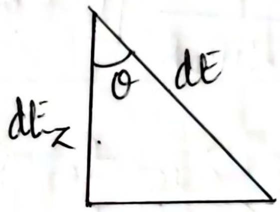
$$= \frac{+\rho_L}{4\pi\epsilon_0 R}\left[+\cos\theta_1 - \cos(\pi-\theta_2)\right].$$

$$\vec{E}_y = \frac{+\rho_L}{4\pi\epsilon_0 R}\left[+\cos\theta_1 + \cos\theta_2\right].$$

$$\vec{E}_y = E_y\,\hat{a}_y$$

$$\boxed{\vec{E}_y = \frac{\rho_L}{4\pi\epsilon_0 R}(\cos\theta_1 + \cos\theta_2)\,\hat{a}_y} \quad \text{— (10)}$$

→ Now let us consider



$$dE_z = dE\cos\theta.$$

$$= \frac{dq}{4\pi\epsilon_0 r^2}\cos\theta.$$

$$E_z = \int \frac{dq}{4\pi\epsilon_0 r^2}\cos\theta.$$

$$E_z = \int_{\pi-\theta_2}^{\theta_1} \frac{\rho_L\,dl\,\cos\theta}{4\pi\epsilon_0 r^2}.$$

$$= \int_{\pi-\theta_2}^{\theta_1} \frac{\rho_L(-R\csc^2\theta\,d\theta)\,\cancel{\sin\theta}\,\cos\theta}{4\pi\epsilon_0(R\csc\theta)^2}$$

$$= -\frac{\rho_L}{4\pi\epsilon_0 R}\int_{\pi-\theta_2}^{\theta_1}\cos\theta\,d\theta = \frac{-\rho_L}{4\pi\epsilon_0 R}(\sin\theta)_{\pi-\theta_2}^{\theta_1}.$$

$$= \frac{-\rho_L}{4\pi\epsilon_0 R}\left[\sin\theta_1 - \sin(\pi-\theta_2)\right]$$

$$= \frac{-\rho_L}{4\pi\epsilon_0 R}(\sin\theta_1 - \sin\theta_2).$$

$$\boxed{dE_z = \frac{\rho_L}{4\pi\epsilon_0 R}(\sin\theta_2 - \sin\theta_1)}$$

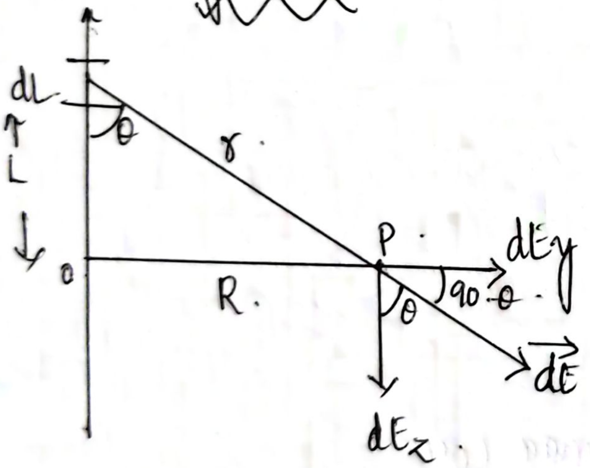
$$\vec{dE_z} = dE_z(-\hat{a}_3)$$

$$\boxed{\vec{dE_z} = \frac{\rho_L}{4\pi\epsilon_0 R}(\sin\theta_1 - \sin\theta_2)\hat{a}_z} \quad -(11)$$

adding equation (10) & (11). we get

$$\vec{E} = \frac{\rho_L}{4\pi\epsilon_0 R}\left[(\cos\theta_1 + \cos\theta_2)\hat{a}_y + (\sin\theta_1 - \sin\theta_2)\hat{a}_z\right]$$

Electric field due to infinitely long conductor of charge density ρ_L C/m.



Let us place the conductor along z-axis.
Electric field intensity at P due to elemental charge dq is

$$\vec{dE} = \frac{dq}{4\pi\epsilon r^2}\hat{a}_r$$

$\vec{dE}$ can be resolved into two rectangular components $\vec{dE_y}$ and $\vec{dE_z}$.

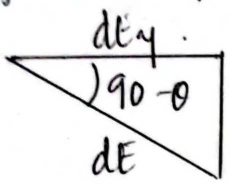
$$\vec{dE} = \vec{dE_y} + \vec{dE_z}$$

Due to symmetry the electric field in the z-direction is zero.

∴ only y component exists.

$$\vec{dE_y} = dE_y\,\hat{a}_y$$

from fig,



$\cos(90^\circ-\theta) = \frac{dE_y}{dE}$

$$dE_y = dE \sin\theta$$

$$dE_y = \frac{dq}{4\pi\epsilon_0 r^2}\sin\theta$$

W.K.T, $dq = \rho_L dL$

$$dE_y = \frac{\rho_L dL}{4\pi\epsilon_0 r^2}\sin\theta$$

from fig,

$$\cot\theta = \frac{L}{R} \Rightarrow L = R\cot\theta$$

$$\boxed{dL = -R\operatorname{cosec}^2\theta \, d\theta}$$

$$\boxed{r = R\operatorname{cosec}\theta}$$

$$\therefore dE_y = \frac{\rho_L(-R\operatorname{cosec}^2\theta \, d\theta)}{4\pi\epsilon_0 (R\operatorname{cosec}\theta)^2}\cdot\sin\theta$$

$$dE_y = \frac{-\rho_L}{4\pi\epsilon_0 R}\left[\sin\theta \, d\theta\right]$$

$$E_y = \frac{-\rho_L}{4\pi\epsilon_0 R}\int_{\theta_1 = \pi-\theta_2}^{\theta_1}\sin\theta \, d\theta$$

$$= \frac{-\rho_L}{4\pi\epsilon_0 R}(-\cos\theta)_{\pi-\theta_2}^{\theta_1}$$

$$= \frac{+\rho_L}{4\pi\epsilon_0 R}(\cos\theta - \cos\theta)$$

$$= \frac{\rho_L}{4\pi\epsilon_0 R}\left[\cos\theta_1 - \cos(\pi-\theta_2)\right]$$

$$E_y = \frac{\rho_L}{4\pi\epsilon_0 R}\left[\cos\theta_1 + \cos\theta_2\right]$$

Since the line charge extends from $-\infty$ to ∞.

At point A, $\theta_2 = 0$.

At point B, $\theta_1 = 0$.

$$\therefore \vec{E} = \frac{\rho_L}{4\pi\epsilon_0 R}\left(\cos 0 + \cos 0\right)\hat{a}_y$$

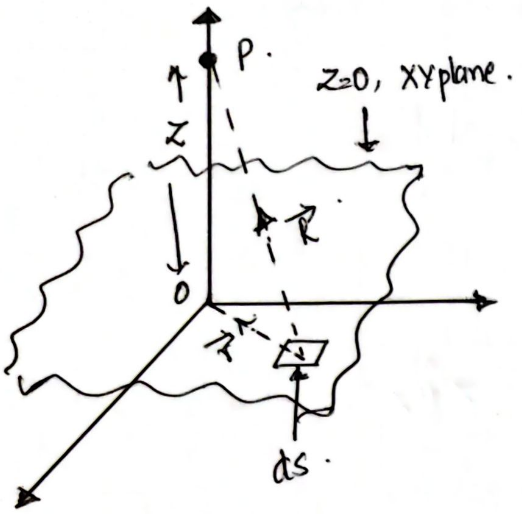
$$= \frac{\rho_L}{4\pi\epsilon_0 R}(2)$$

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 R}\hat{a}_y}$$

Note :- The electric field intensity $\vec{E}$ at any point has no component in the direction parallel to the line along which the charge is located when the charge is infinite.

Eg:- If line charge is parallel to z-axis $\vec{E}$ cannot have $\hat{a}_z$ component, if line charge is parallel to y-axis $\vec{E}$ cannot have $\hat{a}_y$ component.

Electric field due to infinite sheet of charge.



→ Consider an infinite sheet of charge having uniform charge density ρ_s (C/m²) placed in XY plane as shown in fig.

→ Let us use cylindrical coordinates

→ The point P at which $\vec{E}$ to be calculated on z axis

→ Consider the differential surface area 'ds' carrying a charge dQ.

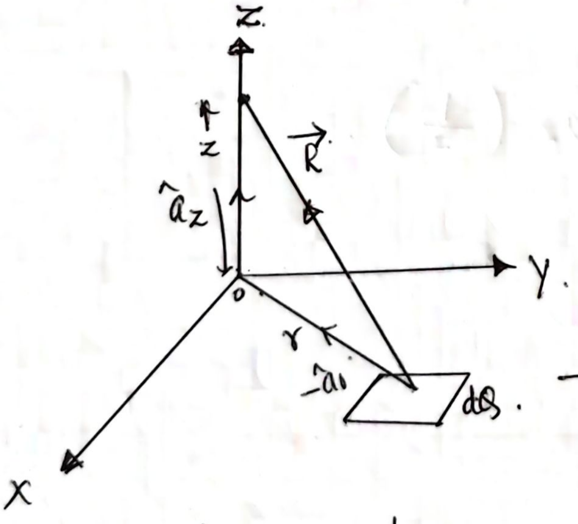
→ "ds" is normal to z-direction

$$\therefore \quad ds = r\,dr\,d\phi$$

$$dQ = \rho_s\, ds$$

$$dQ = \rho_s\, r\,dr\,d\phi$$

$$\vec{dE} = \frac{dQ}{4\pi\epsilon_0 R^2}\hat{a}_R = \frac{\rho_s r\,dr\,d\phi}{4\pi\epsilon_0 R^2}\hat{a}_R$$



Here, $\vec{R} = -r\hat{a}_r + z\hat{a}_z$

$$|\vec{R}| = \sqrt{r^2+z^2}$$

$$\hat{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{-r\hat{a}_r + z\hat{a}_z}{\sqrt{r^2+z^2}}$$

$$\vec{dE} = \frac{\rho_s r\,dr\,d\phi}{4\pi\epsilon_0 (\sqrt{r^2+z^2})^2}\left[\frac{-r\hat{a}_r + z\hat{a}_z}{\sqrt{r^2+z^2}}\right] \quad -(1)$$

'r' varies from 0 to ∞.

'φ' varies from 0 to 2π

$\therefore$ Integrating $\vec{dE}$, eq (1)

$$\vec{E} = \int_{\phi=0}^{2\pi}\int_{r=0}^{\infty}\vec{dE} = \int_{\phi=0}^{2\pi}\int_{r=0}^{\infty}\frac{\rho_s r\,dr\,d\phi}{4\pi\epsilon_0 (r^2+z^2)^{3/2}}(z\hat{a}_z)$$

→ As there is symmetry about z axis from all radial direction, all $\hat{a}_r$ components of $\vec{E}$ are going to cancel each other & net $\vec{E}$ will not have any radial component.

→ put $r^2 + z^2 = t^2$ and $2r\,dr = 2t\,dt$

for $r = 0, \quad t = z$.

$r = \infty, \quad t = \infty$.

$$\vec{E} = \int_0^{2\pi} \int_{t=z}^{\infty} \frac{\rho_s}{4\pi\epsilon_0} \cdot \frac{t\,dt}{(t^2)^{3/2}} \, d\phi \cdot z\hat{a}_z$$

$$= \int_0^{2\pi} \int_{t=z}^{\infty} \frac{\rho_s}{4\pi\epsilon_0} \frac{dt}{t^2} \, d\phi \, (z\hat{a}_z).$$

$$= \int_0^{2\pi} \frac{\rho_s}{4\pi\epsilon_0} \cdot d\phi \, z\hat{a}_z \left(\frac{-1}{t}\right)_z^{\infty}$$

$$= \frac{\rho_s}{4\pi\epsilon_0} [\phi]_0^{2\pi} \, z\hat{a}_z \left[-\frac{1}{\infty} - \left(\frac{-1}{z}\right)\right].$$

$$= \frac{\rho_s}{4\pi\epsilon_0} \times 2\pi \times z\hat{a}_z \left(\frac{1}{z}\right).$$

$$\boxed{\vec{E} = \frac{\rho_s}{2\epsilon_0} \hat{a}_z \ \text{N/m}}$$

Electric flux

The total number of lines of force in any particular electric field is called the electric flux. It is represented by symbol ψ. Its unit is C.

properties

1. The flux lines start from the +ve charge & terminate on the -ve charge.
2. If the -ve charge is absent, then the flux lines terminate at infinity. while in absence of the +ve charge the electric flux terminates on the -ve charge from infinity.
3. The density of flux lines over a certain region is proportional to the strength of the field in the same region. ie there will be crowding of lines at a stronger field region.
4. If the charge on a body is $\pm Q$ C, then the total number of lines originating or terminating on it is also Q.

ie $\boxed{\psi = Q}$

flux is a scalar field. Vector field associated with the flux is called electric flux density

Electric flux Density ($\vec{D}$).

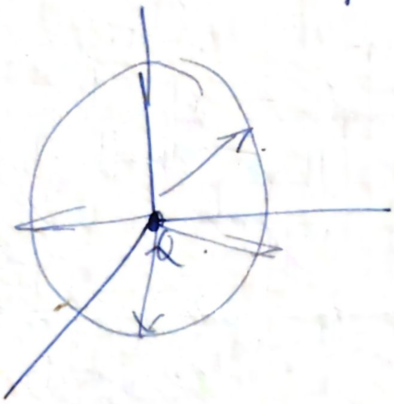
Consider two point charges as in fig.

The flux lines originates from +ve charge & terminate at -ve charge.

Consider a unit surface area as in fig. The net flux passing normal through the unit surface is called the E.F.D. It has a specific direction which is normal to the surface area under consideration hence it is a vector field.

$$\therefore \vec{D} = \frac{\psi}{S} \hat{a}_n$$

Consider a pt charge +Q placed at the centre of the imaginary sphere of radius r. as in fig.



The flux lines originating from the point charge +Q are directly radiated outwards.

$\therefore$ E.F.D

$$\vec{D} = \frac{\text{total flux}}{\text{total surface area}}$$

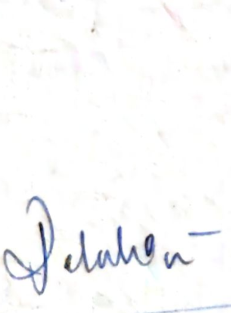
$$\psi = Q$$

$$S = 4\pi r^2$$

$$\vec{D} = \frac{Q}{4\pi r^2} \hat{a}_r$$

$$|\vec{D}| = \frac{Q}{4\pi r^2} \ C/m^2$$

Relation b/w $\vec{D}$ & $\vec{E}$



Consider an imaginary sphere with charge Q as centre and 'r' as its radius.

The E.F.I at P is

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{a}_r \quad -(1)$$

The E.F.D. is

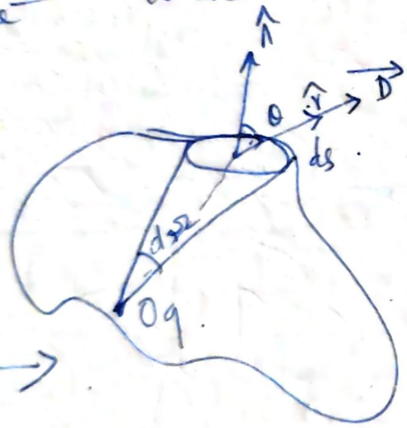
$$\vec{D} = \frac{Q}{4\pi r^2}\hat{r} \quad -(2).$$

Comparing (1) & (2).

$$\boxed{\vec{D} = \epsilon_0 \vec{E}}$$

Gauss Law

It states that "The total electric flux passing thro' any closed surface is equal to the total charge within the surface".



Consider a closed surface of any arbitrary size which encloses the charge q. Consider a differential surface ds at point P.

The flux $d\psi$ leaving the surface element ds is given by

$$d\psi = \vec{D}\cdot\vec{ds}$$

$$= \frac{q}{4\pi r^2}\hat{r}\cdot ds\,\hat{n}$$

[Let $\hat{n}$ be the unit vector erected normal to the elemental area ds] $\vec{ds} = ds\,\hat{n}$

where $\hat{r}\cdot\hat{n} = 1.1.\cos\theta = \underline{\underline{\cos\theta}}$

Total flux

$$\psi = \oint_s d\psi = \oint_s \vec{D}\cdot\vec{ds} \cancel{= Q}.$$

9. In a certain region of space, $\vec{D} = 2xy\hat{a}_x + 3yz\hat{a}_y + 4zx\hat{a}_z$. Evaluate the amount of electric flux that passes through the portion bounded by $-1 \le y \le 2$ & $0 \le z \le 4$ in the $x = 3$ plane.

:- $\vec{D} = 2xy\hat{a}_x + 3yz\hat{a}_y + 4xz\hat{a}_z$.

Plane is parallel to y-z plane and intercepting the x axis at $x = 3$.

$\therefore \vec{ds} = dy\,dz\,\hat{a}_x$

$$d\psi = \vec{D}\cdot\vec{ds} = [2xy\hat{a}_x + 3yz\hat{a}_y + 4xz\hat{a}_z)(dy\,dz\,\hat{a}_x)$$

$$d\psi = 2xy\,dy\,dz.$$

$$\psi = \int d\psi = \int_{z=0}^{4}\int_{y=-1}^{2} 2xy\,dy\,dz.$$

$$= \int_{z=0}^{4}\left[2x\,y^2/2\right]_{-1}^{2} dz = \int_{0}^{4} x\,(y^2)_{-1}^{2}\,dz = 3x\int_{0}^{4} dz$$

$$= 3x\,(z)_0^4 = 12x.$$

$x = 3$

$$\therefore \psi = 12 * 3 = \boxed{36\ C}$$

10. Given a 60 μC point charge located at the origin, find the total electric flux passing through a) that portion of the sphere $r = 26$ cm bounded by $0 < \theta < \pi/2$ & $0 < \phi < \pi/2$ b) the closed surface defined by $\rho = 26$ cm & $z = \pm 26$ cm. (c) the plane $z = 26$ cm.

:- a) total flux

$$\psi = \int \vec{D} \cdot \vec{ds}$$

where $\vec{D} = \frac{Q}{4\pi r^2} \hat{a}_r$

$$\vec{ds_r} = r^2 \sin\theta \, d\theta \, d\phi \; \hat{a}_r$$

$$\therefore \vec{D} \cdot \vec{ds_r} = \frac{Q}{4\pi r^2} \hat{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi \; \hat{a}_r$$

$$= \frac{Q}{4\pi} \sin\theta \, d\theta \, d\phi$$

$$\psi = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{\pi/2} \frac{Q}{4\pi} \sin\theta \, d\theta \, d\phi$$

$$= \frac{Q}{4\pi} \int_{\theta=0}^{\pi/2} \sin\theta \, d\theta \, [\phi]_0^{\pi/2} = \frac{Q}{4\pi} \int_{\theta=0}^{\pi/2} \frac{\pi}{2} \sin\theta \, d\theta$$

$$= \frac{Q}{8} \int_{\theta=0}^{\pi/2} \sin\theta \, d\theta = \frac{Q}{8} \left[-\cos\theta\right]_0^{\pi/2} = +\frac{Q}{8}[0+1]$$

$$= \frac{60 \times 10^{-6}}{8} = \underline{\underline{7.5 \times 10^{-6} \text{ C}}}$$

b) $\psi = \int \vec{D} \cdot \vec{ds}$ $\qquad \vec{ds_r} = r \, d\phi \, dz \; \hat{a}_r$

$$\vec{D} = \frac{Q}{4\pi r^2} \hat{a}_r$$

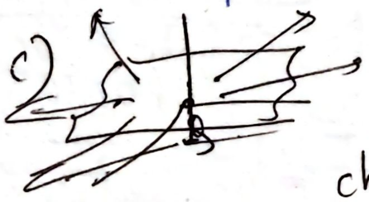
$$\psi = \int \frac{Q}{4\pi r^2}\hat{a}_r \cdot r\, d\phi\, dz\, \hat{a}_r$$

$$= \frac{Q}{4\pi r}\int d\phi\, dz = \frac{Q}{4\pi r}\int_{\phi=0}^{2\pi}\int_{z=-26}^{26} d\phi\, dz$$

$$= \frac{Q}{4\pi r}\int_{\phi=0}^{2\pi}[z]_{-26}^{26}\, d\phi = \frac{Q}{4\pi r}\int_{\phi=0}^{2\pi} 52\, d\phi$$

$$= \frac{Q}{4\pi r} * 52 * 2\pi = \frac{60\times10^{-6}}{4\pi * 26\times10^{-2}}\times 52\times 2\pi$$

$$= 6\times10^{-3}\ C$$

c) 

As plane is ∞, half the total flux originating from charge will pass through $\psi = Q/2$

(11). Calculate D in rectangular co-ordinates at point P(2, -3, 6) produced by a) a point charge $Q_A = 55\ mC$ at Q(-2, 3, -6) (b) a uniform line charge $\rho_{LB} = 20\ mC/m$ on the x axis (c) a uniform surface charge density $\rho_{sc} = 120\ \mu C/m^2$ on the plane $z = -5m$.

Sol — a) $\vec{D} = \frac{Q}{4\pi r^2}\hat{a}_r$

$$= \frac{55\times10^{-3}}{4\pi * 14^2}\left\{\frac{(2+2)\hat{a}_x + (-3-3)\hat{a}_y + (6+6)\hat{a}_z}{14}\right\}$$

$$= 6.38\,\hat{a}_x - 9.57\,\hat{a}_y + 19.14\,\hat{a}_z\ \mu C/m^2$$

b) $\vec{D} = \frac{\cancel{\phi}}{\cancel{4\pi r^2}} \hat{a}_r$

$= \frac{20 \times 10^{-3}}{4\pi \times 45} \frac{(-3\hat{a}_y + 6\hat{a}_z)}{\sqrt{45}}$

$\vec{D} = \frac{\rho_L}{2\pi r} \hat{a}_r$

$= \frac{20 \times 10^{-3}}{2\pi \times \sqrt{45}} \left\{ \frac{-3\hat{a}_y + 6\hat{a}_z}{\sqrt{45}} \right\}$

$= -212\hat{a}_y + 424\hat{a}_z \; \mu C/m^2$

c) $\vec{D} = \frac{\rho_s}{2} \hat{a}_z = \frac{120 \times 10^{-6}}{2} \hat{a}_z = 60 \mu C/m^2$

10) c) $\psi = \int \vec{D} \cdot \vec{ds}$

$\vec{D} = \frac{Q}{4\pi r^2} \hat{a}_r \qquad \vec{ds}_z = r\,dr\,d\phi\,\hat{a}_z$

$= \frac{Q}{4\pi} \int_{r=0}^{10} \int_{\phi=0}^{2\pi} \frac{1}{r}\, dr\, d\phi$

$= \frac{Q}{4\pi}$

(12) If $\vec{D} = 9x^2\hat{a}_x + 5y^2\hat{a}_y + 2z\hat{a}_z$ c/m², find the charge density at the point $(1, 5, 9)$ m. (25)

:- $\rho_v = \nabla \cdot \vec{D}$.

$$= \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}.$$

$$\rho_v = 27x^2 + 10y + 2$$

$$\rho_{v(p)} = 79 \ c/m^3.$$

(13) A 25μC charge is located at the origin. Calculate the total electric flux passing through the portion of the surface.

i) with $r = 0.2m$, $0 < \theta < \pi$, $0 < \phi < \pi/2$.

ii) closed surface with $\rho = 0.8m$ & $z = \pm 0.5m$.

:- i). $Q = \psi = \int \vec{D} \cdot \vec{ds}$.

$$\vec{D} = \frac{Q}{4\pi r^2}\hat{a}_r \quad \frac{\cancel{25 \times 10^{-6}}}{}$$

$$\vec{ds}_r = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

$$= \int_\theta \int_\phi \frac{Q}{4\pi r^2}\hat{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r = \int_{\theta=0}^{\pi}\int_{\phi=0}^{\pi/2} \frac{Q}{4\pi} \sin\theta \, d\theta \, d\phi.$$

$$(\because \hat{a}_r \cdot \hat{a}_r = 1).$$

$$= \frac{Q}{4\pi}\int_{\theta=0}^{\pi} \sin\theta \, d\theta \, [\phi]_0^{\pi/2} = \frac{Q}{4\pi}\int_{\theta=0}^{\pi} \sin\theta \, d\theta \, (\pi/2 - 0)$$

$= \frac{Q}{8} \int_{\theta=0}^{\pi} \sin\theta \, d\theta = \frac{Q}{8} \left[-\cos\theta\right]_0^{\pi}$

$= \frac{Q}{8} [1+1] = \frac{25 \times 10^{-6}}{8} * 2 = \boxed{6.25 \times 10^{-6} C}$

4. ii) closed cylindrical surface.

$\psi = \oint_s \vec{D} \cdot \vec{ds}$

$= \int_{\rho} \int_{\phi} \int_{z} \vec{D} \cdot \vec{ds}$

$\vec{D} = \frac{Q}{4\pi r^2} \hat{a}_r \qquad \vec{ds}_{\rho} = r \, d\phi \, dz \, \hat{a}_r$

$\psi = \iiint_{r \phi z} \frac{Q}{4\pi r^2} \hat{a}_r \cdot r \, d\phi \, dz \, \hat{a}_r$

$= \frac{Q}{4\pi r} \int_{\phi=0}^{2\pi} \int_{z=-0.5}^{0.5} d\phi \, dz$

$= \frac{Q}{4\pi r} [\phi]_0^{2\pi} [z]_{-0.5}^{0.5}$

$\frac{Q}{4\pi r} [2\pi - 0) (0.5 + 0.5) = \frac{Q}{2r} = \frac{25 \times 10^{-6}}{2 \times 0.8}$

?

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(4). Given $\vec{D} = r/3 \ \hat{a}_r \ C/m^2$ (26)

i) find the E.F.I at $r = 0.2m$.

ii). total charge within the sphere $r = 3m$.

iii). total electric flux leaving the sphere at $r = 0.4m$.

:- i) $\vec{E} = \cancel{\epsilon_0 \vec{D}}$.

$\vec{D} = \epsilon_0 \vec{E}$.

$$\vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{r/3 \ \hat{a}_r}{8.85 \times 10^{-12}} = \frac{0.2}{3 \times 8.85 \times 10^{-12}} \hat{a}_r$$

$$= \boxed{7.52 \times 10^{9}} \ \hat{a}_r \ V/m.$$

ii) $\vec{D} = \frac{Q}{4\pi r^2}$.

$$\Rightarrow Q = \vec{D} . 4\pi r^2$$

$$= \frac{r}{3} \hat{a}_r . 4\pi r^2 = \frac{3}{3} \hat{a}_r . 4\pi . 3^2$$

$$= \underline{\underline{113.04}} \ C.$$

iii) $\vec{D} = \frac{Q}{4\pi r^2} = \frac{\Psi}{4\pi r^2}$

$$\Psi = \vec{D} . 4\pi r^2.$$

$$= \frac{r \cancel{Q}}{3 \cancel{4\pi r^2}} \hat{a}_r . 4\pi r^2 = \frac{0.4}{3} \times 4\pi \times (0.4)^2 \hat{a}_r$$

$$= 267.94 \times 10^{-3} \ C$$

(15) Find the total flux passing through the sphere of radius 2m. Given that $\vec{D} = 1/r^2 \, \hat{a}_r$ C/m².

:- $\psi = \vec{D} \cdot 4\pi r^2$

$= \frac{1}{r^2} \hat{a}_r \cdot 4\pi r^2$

$= \frac{1}{4} \hat{a}_r \cdot 4\pi \cdot 4 = 4\pi \hat{a}_r = \underline{12.56 \hat{a}_r \text{ C/m}^2}$

(16). Given $\vec{D} = 0.3 r^2 \hat{a}_r$ nC/m² in free space.

i) Find $\vec{E}$ at $r = 2m$, $\theta = 25°$, $\phi = 90°$.

ii) Find the total charge within the sphere $r = 3$.

iii). Find the total electric flux leaving the sphere $r = 4$.

:- i) $\vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{0.3 * 2^2 \times 10^{-9} \hat{a}_r}{8.85 \times 10^{-12}}$

$= \underline{\underline{135.5 \hat{a}_r \text{ V/m}}}$

ii) $\vec{D} = \frac{Q}{4\pi r^2} \Rightarrow Q = \vec{D} \cdot 4\pi r^2$

$= 0.3 * 3^2 * 10^{-9} \times 4\pi \times 3^2$

$= \underline{\underline{305 \text{ nC}}}$

iii) $\vec{D} = \frac{Q}{4\pi r^2} = \frac{\psi}{4\pi r^2}$

$\psi = \vec{D} \cdot 4\pi r^2 = 0.3 \times 4^2 \times 10^{-9} \times 4\pi \times 4^2$

$= \underline{\underline{965 \text{ nC}}}$

(17). Calculate the total charge within each of the indicated volumes. (27)

i) $|x| < 0.1$, $|y|, |z| \le 0.2$ $\rho_v = \dfrac{1}{x^3 y^3 z^3}$

ii) $0 \le r < 0.1$, $0 \le \phi \le \pi$, $-2 \le z \le 4$, $\rho_v = r^2 z^2 \sin(0.6\phi)$.

iii) universe $\rho_v = \dfrac{e^{-2r}}{r^2}$.

:- i). $\psi = Q = \int_v \rho_v dv$.

$$= \int \frac{1}{x^3 y^3 z^3} dx dy dz = \int_{-0.1}^{0.1} x^{-3} dx \int_{-0.2}^{0.2} y^{-3} dy \int_{-0.2}^{0.2} z^{-3} dz.$$

$$= \left[\frac{x^{-2}}{-2}\right]_{-0.1}^{0.1} \left[\frac{y^{-2}}{-2}\right]_{-0.2}^{0.2} \left[\frac{z^{-2}}{-2}\right]_{-0.2}^{0.2} = 0.$$

ii). $Q = \int \rho_v dv$.

$$= \int r^2 z^2 \sin(0.6\phi)\, r dr d\phi dz.$$

$$= \int_0^{0.1} r^3 dr . \int_0^{\pi} \sin(0.6\phi) d\phi . \int_2^4 z^2 dz.$$

$$= \left[\frac{r^4}{4}\right]_0^{0.1} \left[-\frac{\cos(0.6\phi)}{0.6}\right]_0^{\pi} \left[\frac{z^3}{3}\right]_2^4.$$

$$= \frac{1}{4}(100 \times 10^{-6}) \times \frac{1}{0.6} \left[\right.$$

$$= 1.018 \, mC \qquad 2.5 \times 10^{-5} \times \frac{(0.809 + 1)}{0.6} \times$$

iii) The universe can be considered as a spherical volume for which the radius is very large & the limits for r, θ, ϕ

are 0 to ∞, 0 to π & 0 to 2π.

$$Q = \int \rho_v \, dv$$

$$= \int \frac{e^{-2r}}{r^2} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \int_{r=0}^{\infty} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{e^{-2r}}{\cancel{r^2}} \cancel{r^2} \sin\theta \, dr \, d\theta \, d\phi$$

$$= \int_0^{\infty} e^{-2r} dr \int_{\theta=0}^{\pi} \sin\theta \, d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$= \left[\frac{e^{-2r}}{-2}\right]_0^{\infty} \left[-\cos\theta\right]_0^{\pi} \left[\phi\right]_0^{2\pi}$$

$$= +\frac{1}{2}(0-1)(-1-1)(2\pi - 0)$$

$$= -\frac{1}{2}(-2)(2\pi) = 2\pi \quad = \underline{\underline{6.28C}}.$$

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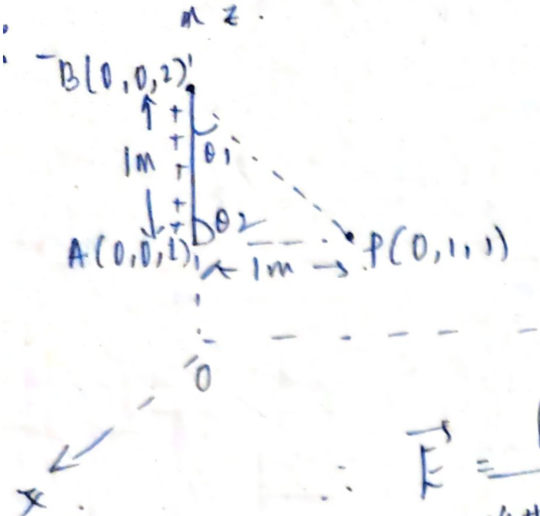
(18) A line charge of total charge $2\mu c$ is located b/w A (0,0,1) & B (0,0,2) m. Find the electric field intensity at P (0,1,1) m.



$\rho_L = 10^{-6}$ C/m.

$\theta_1 = 45^\circ$ ($\because$ ABP is an isosceles rt. angled triangle)

$\theta_2 = 90^\circ$.

$\therefore \vec{E} = \dfrac{\rho_L}{4\pi\epsilon_0 R}\left[(\cos\theta_2 + \cos\theta_1)\hat{a}_y + (\sin\theta_2 - \sin\theta_1)\hat{a}_z\right]$

$\vec{E} = (6363\,\hat{a}_y + 2637\,\hat{a}_z)$ V/m.

4) A uniform line charge of 250 nC/m is located at $x_1 = -3$, & $z_1 = 4$ in free space. Find the E.F.I at (2,5,3)

The line charge is $\parallel$le to y-axis.

$\therefore \vec{E} = \dfrac{\rho_L}{2\pi\epsilon_0 r}\hat{a}_r$

$= \dfrac{250\times10^{-9}}{2\pi\times8.85\times10^{-12}\times}\left\{\dfrac{(2+3)\hat{a}_x + (3-4)\hat{a}_z}{\sqrt{5^2+(-1)^2}}\right\}$

$= \dfrac{250\times10^{-9}}{2\pi\times8.85\times10^{-12}\times5.1}(5\hat{a}_x - \hat{a}_z)$

$= 172\,(5\hat{a}_x - \hat{a}_z)$.

$|\vec{E}| = 877.03$ V/m.

(29) An infinite line charge with $\rho_L = 30\,nC/m$ is located at $y=3$ & $z=5$. Find the E.F.I at
a) origin b) (0,6,1) c) (5,6,1)

:- The ~~th~~ line charge is ||le to x-axis.

∴ $y_1 = 3$ $y_2 = 0$

$z_1 = 5$ $z_2 = 0$.

a) $$\bar{E} = \frac{\rho_L}{2\pi\epsilon d}\left\{\frac{(0-3)\hat{a}_y + (0-5)\hat{a}_z}{\sqrt{9+25}}\right\}$$

$$= \frac{-30\times10^{-9}}{2\pi\times8.85\times10^{-12}\times34}(3\hat{a}_y+5\hat{a}_z) = -47.604\hat{a}_y - 79.34\hat{a}_z$$

$$|\bar{E}| = 92.525\ V/m$$

(b) ~~$\bar{E} = 64.74\hat{a}_n$~~

$$\bar{E} = \frac{+30\times10^{-9}}{2\pi\times8.85\times10^{-12}}\frac{(3\hat{a}_y - 4\hat{a}_z)}{25}.$$

$$= 21.59(3\hat{a}_y - 4\hat{a}_z).$$

(c)

(21) A uniform line charge density $\rho_L = 25$ nC/m on the line $x = -3$ m, $y = 0$ & $z = 4$ m in free space. Find the E.F.I at a point $(2, 15, 3)$ mete

:- The line charge is $\parallel^{le}$ to y-axis

$$\therefore \vec{E} = \frac{\rho_L}{2\pi\epsilon R}\hat{a}_L = \frac{25\times10^{-9}}{2\pi\times8.85\times10^{-12}}\left\{\frac{(2+3)^2\hat{a}_x + (3-4)\hat{a}_z}{5.1}\right\}$$

$$\vec{E} = 86.39\,\hat{a}_x - 17.28\,\hat{a}_z \text{ V/m}$$

$$|\vec{E}| = 88.1 \text{ V/m}$$

(22) A line charge of 20nC/m is located at $x = 6$, $y = 8$. Determine the E.F.I at $(5, 4, 2)$

:- The line charge is $\parallel^{le}$ to z axis.

$$\therefore \vec{E} = \frac{20\times10^{-9}}{2\pi\times8.85\times10^{-12}}\left\{\frac{(5-6)\hat{a}_x + (4-8)\hat{a}_y}{17}\right\}$$

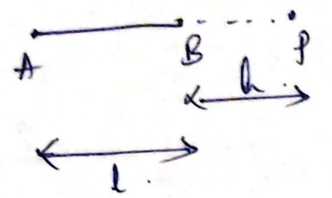
$$= -21.14\,\hat{a}_x - 84.7\,\hat{a}_y$$

$$|\vec{E}| = 87.29 \text{ V/m}.$$

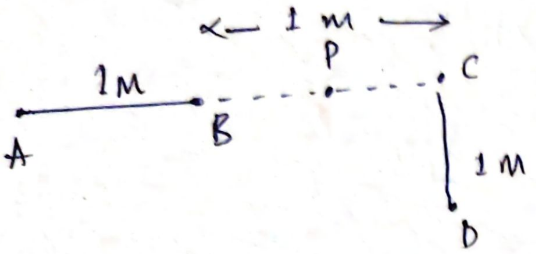
(23) Two wires AB & CD, each 1 m length, carry a total charge of $0.2\mu C$ each & are arranged as shown. The ends B & C are separated by 1 m distance. Determine the value of E at the point P in the vector form C, P is the mid point of BC.

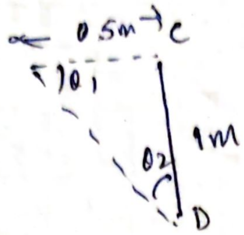
NOTE :- When the point P is along the line charge at a distance 'h' from the right end.

The electric field due to line charge of length AB at point P is



$$E = \frac{\rho_L}{4\pi\epsilon}\left[\frac{1}{h} - \frac{1}{l+h}\right]$$





$R = 0.5$

$$\vec{E}_2 = \frac{\rho_L}{4\pi\epsilon_0 R}\left[(\cos\theta_2 + \cos\theta_1)\hat{a}_y + (\sin\theta_2 - \sin\theta_1)\hat{a}_z\right]$$

$$= \frac{0.2\times10^{-6}}{4\pi\times8.85\times10^{-12}\times0.5}\left[\cos(26.57 + \cos 63.43)\hat{a}_y + (\sin 26.57 - \sin 63.43)(-\hat{a}_z)\right]$$

$$= 4821\,\hat{a}_y + 1607\,\hat{a}_z$$

$\theta_1 = \tan^{-1}(2) = 63.43$

$\theta_2 = 90 - 63.43 = 26.57$

$$\vec{E}_1 = \frac{0.2\times10^{-6}}{4\pi\times8.85\times10^{-12}}\left[\frac{1}{0.5} - \frac{1}{1.5}\right] = 2400 \text{ V/m}$$

$$\vec{E} = 4821\,\hat{a}_y + 1607\,\hat{a}_z$$

0°	30°	45°	60°	90°
0	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$
0	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1
1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	0